1. Build 6 square macro-modules. Assemble according to the cube structure (macromodular technique).



Figure 7 Twirl model, variant 1.

2. Build 8 triangle macro-modules. Assemble according to the octahedron structure (macro-modular technique).



Figure 8 Twirl model, variant 2.

3. Assemble step by step (single module in each step). First make a square rose (from 4 modules), make a ring of squares and triangles around it, next a ring of squares, a second ring of triangles and squares, finally close the model.





Figure 9 Twirl model, variant 3.

Build 2 square macro-modules (roses) and two rings, 8 modules for each ring.
Assemble a square rose with a ring, then assemble two halves into a model.



## Figure 10 Twirl model, variant 4.

5. Build 2 square macro-modules (roses) and a double ring (a ring of squares). Attach roses to both sides of the ring.





Figure 11 Twirl model, variant 5.

Despite different assembly processes all final models have exactly the same structure: a lesser rhombic cuboctahedron. The first two models show also the strong relation between a cube and an octahedron.



Figure 12 The common structure of all variants: a rhombic cuboctahedron.

This is a good example that such mathematical concept as duality may emerge naturally from real life experience.

The examples above show that the same model may be used to present different aspects of symmetry. Different ways to complete the model should reflect the educational goal. In the case of less advanced students or concentration on the model, easier way (variant

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4 or 5) may be selected. If a structure is the main educational goal the way that reflects the structure more deeply may be selected.

As an exercise to the reader, we propose to figure out what will be the final structure of a model built from triangle and square macro-modules that are assembled according to the structure of a lesser rhombic cuboctahedron.

## 4. SYMMETRY AS A GENERATOR OF TWIRL MODELS

## 4.1 How are twirl models built?

Twirl modules use tension of paper to join macro-modules together. It allows some flexibility. There is no rigid restriction on length or angle of a specified part of a module. Because of that, a macro-module technique is very useful to create different twirl models. Diversity of twirl models is based on the following factors (Burczyk, 2008):

- The base module design (the origami part of work).
- Joining modules into different types of macro-modules. Macro-modules are joined in the manner preserving symmetry.
- Joining macro-modules into a finished model based on Platonic or Archimedean polyhedra.

Macro-modules can be build from twirl modules in two basic ways: with single spirals going outward at each base module (in a vertex of a polygon corresponding to the macro-module) and with double spirals going outward at each base module (in a vertex of a polygon corresponding to the macro-module).

There are two possible ways to join macro-modules in the case of a single outward spiral (Figure 13):



Figure 13 Two ways to assemble twirl macro-modules with single outward spiral (Burczyk, 2009) and examples of models assembled in such a way.

There are four possible ways to join macro-modules in the case of double outward spirals (Figure 14):

- edge to edge;
- vertex to edge;
- vertex to vertex;
- a double spiral vortex (a vortex built from two adjacent spirals of a macromodule) to a double spiral vortex.