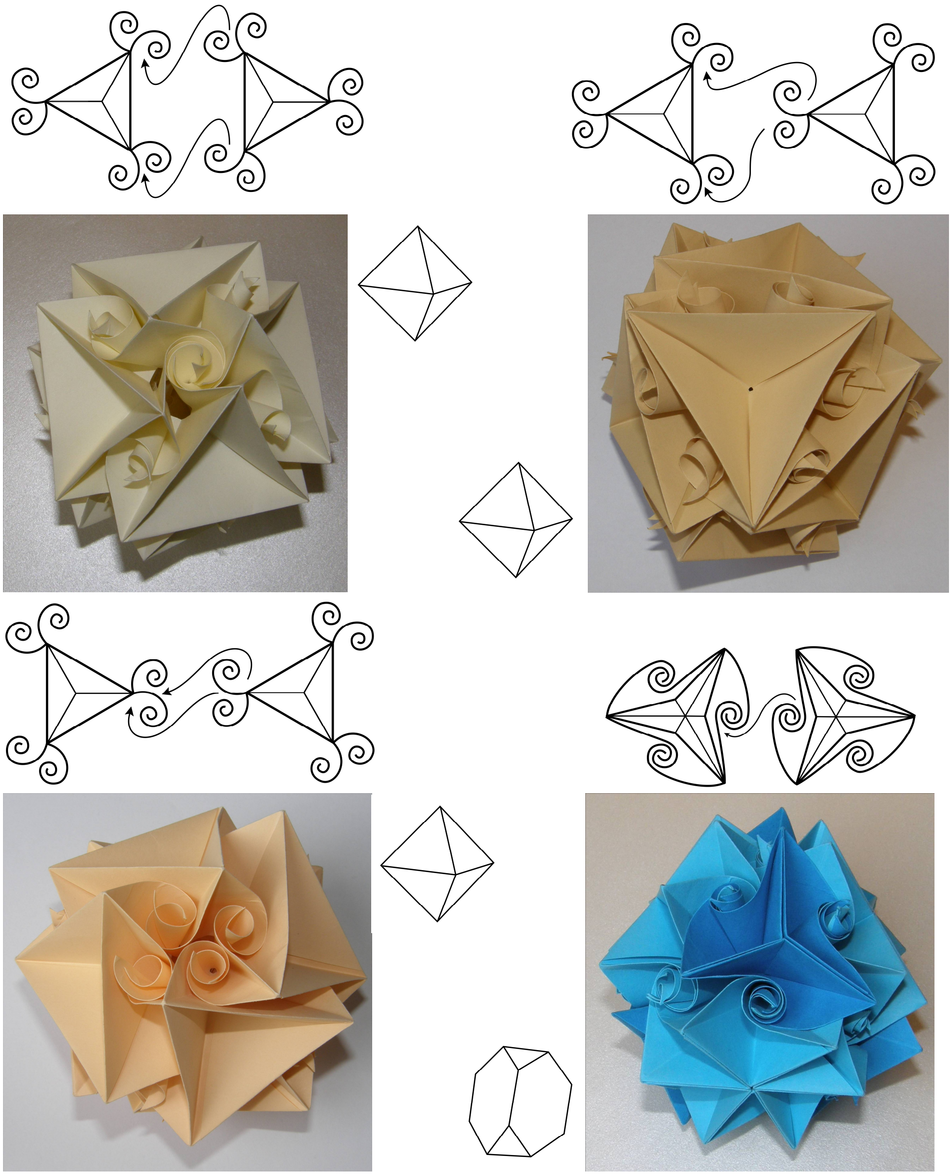

Figure 14 Four ways to assemble twirl macro-modules with double outward spirals and the corresponding models.

Usually every combination of a base module, a joining method and a polyhedron structure produces a stable origami model. Combination of simple rules and a small number of primary elements generates a huge variety of models.

4.2 This one or rather the dual one

Sometimes the correspondence of parts of an origami model and a polyhedron structure is not obvious. Figure 15 shows a triangle macro-modules joined according to the octahedron structure.

A surprise – the finished model looks much more like a cube than an octahedron. Maybe we should interpret a 4-spial vortex as a square face of a cube instead of a 4-fold vertex of an octahedron? Or to get a *proper* polyhedron we should interpret each vortex as a face of cuboctahedron? Which interpretation is the correct one?

Such a model shows that correspondence between the real object (an origami model) and the mathematical structure is much more a matter of convention than we usually expect. It is also a nice educational problem showing that sometimes there is no unique correct answer. In our case, there are three different equally correct answers.

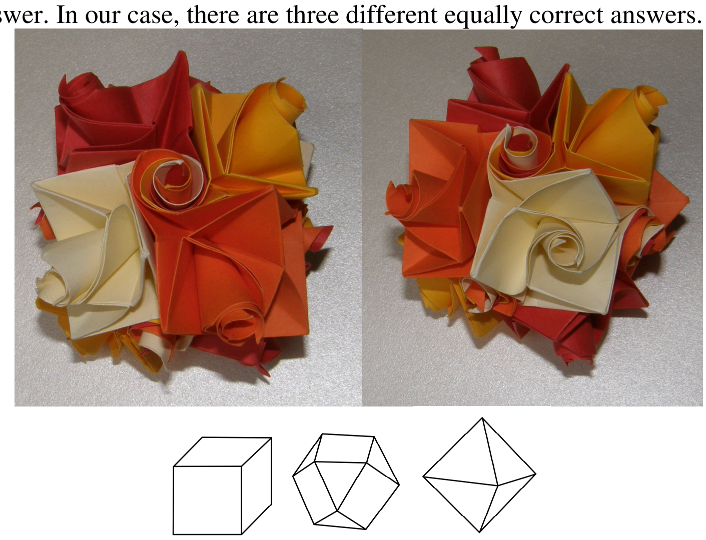

Figure 15 Duality of polyhedra or duality of interpretation?

4.3 The same or different?

Sometimes the appearance of a model hides its structure. Models shown in the figure 16 seem to be the same at first glance. The flower-like spiky appearance hides structural differences. After a second look, you can discover the different structure of the models: a truncated octahedron with squares and hexagons (left) and a dodecahedron with pentagons (right).

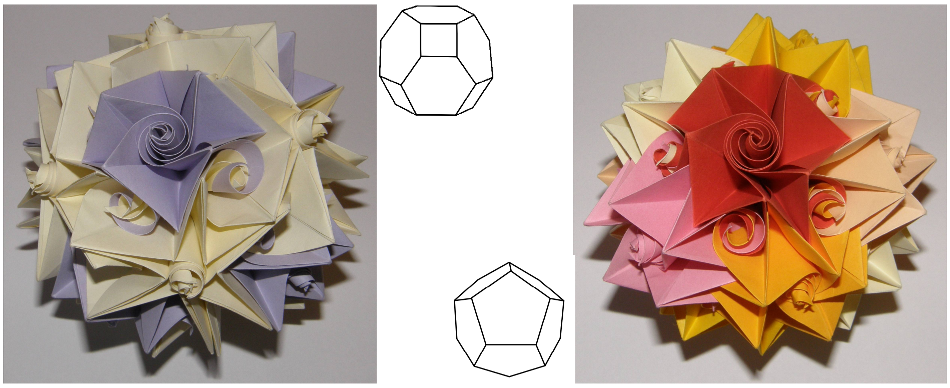

Figure 16 Almost the same appearance but different structure.

4.4 Different or the same?

The opposite may also happen. Despite differences in appearance, models have the same polyhedron structure. In figure 17, both models are based on the same structure: a tetrahedron.

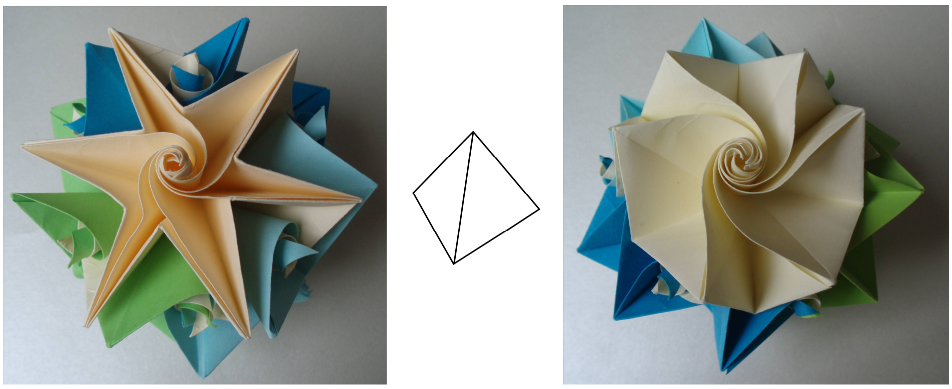

Figure 17 Different appearance but the same structure.

5. ORIGAMI AS AN EDUCATIONAL TOOL

The examples above show how polyhedral structures lead to a variety of visually attractive origami models. Through origami we can easily compose different polyhedral structures. Making origami models leads naturally to questions and problems that can be used in mathematical education. Such activity develops spatial imagination and recognition – elements are manipulated in the space, rotated and translated, different (not only orthogonal) orientation of objects and rotation axes are introduced naturally. Geometrical structure (relations between faces, vertices and edges, sections) can be investigated to fix the geometrical concepts. Relations between different polyhedra can be easily studied. According to our observation, the most difficult part in model making is not the folding but the mathematical part: preserving the symmetry of the model.

Proper use of origami creates a situation when mathematics is hidden but still necessary to achieve the goal. Modular origami gives us a tool for natural introduction of spatial geometry concepts and terms. The concept and terms related to polyhedra, polygons and different types of symmetry in the space are introduced naturally and support the process giving an attractive result. Such terms are not treated as artificial in the context of making a modular origami model contrary to the very frequent situation of introducing such terms in the mathematics teaching process. Introducing names of geometrical objects and properties and mathematical terms in the process of building origami models is accompanied by multi-sensored activities, which enhances their memorization.

6. CONCLUSIONS

1. Symmetry is the key to nice a looking model. If symmetry is preserved, the tension forces of paper work for us and model is stable and regular. Otherwise, forces of paper tend to squeeze the model or the model falls apart.
2. Symmetry is a “generator” of origami models. When origami part is ready – a new base module is invented, the use of polyhedra symmetry generates several origami models “for free”.
3. Making origami models can play an important role in the mathematical education. It develops spatial skills. Symmetry is the necessary part of model construction, often the most difficult part of model assembly. On the one hand, it is impossible to avoid the mathematical aspect of a model construction; on the other, mathematics is hidden behind a real life goal, what creates an environment that supports development of mathematical skills by students.
4. Symmetry is not so easily visible as it may be expected in such a simple activity as origami model folding.

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